Turbulent flow in a stably stratified atmosphere

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SUMMARY

Fluctuations of velocity and temperature which occur in a turbulent flow in a stably-stratified atmosphere far from restraining boundaries are discussed using the equations for the turbulent intensity and for the mean square temperature fluctuation. From these, an equation is derived for the flux Richardson number in terms of the ordinary Richardson number and some non-dimensional ratios connected with the turbulent motion. It is shown that the interaction between the temperature and velocity fields imposes on the flux Richardson number an upper limit of 0·5, and on the ordinary Richardson number a limit of about 0·08. If these values are exceeded, no equilibrium value of the turbulent intensity can exist and a collapse of the turbulent motion would occur. Although the analysis applies strictly only to a homogeneous non-developing flow, it should have approximate validity for effectively homogeneous, developing flows, and the predictions are compared with some recent observations of these flows.

1. INTRODUCTION

The study of turbulent flow in a stably-stratified fluid may be helped by the recognition of some different types of irregular flow which may occur. If the direct influence of the buoyancy forces on the motion is small, the motion will resemble 'ordinary' turbulence and will be characterized by high rates of energy dissipation, and of momentum and mass transport. If the buoyancy forces are dominant, the motion may be an irregular collection of gravity waves with low rates of energy dissipation and transport. In flows unrestrained by boundaries, both types of motion are found, characteristically turbulent motion near the origin of the flow and wave-like motion far downstream. There is here a gradual transition from the region of turbulent flow to the region of wave flow, which makes difficult the definition of the limits of either flow; nevertheless, the two flows are so distinct that no common description is likely to be valid.

In this paper, the characteristically turbulent flow in a stably-stratified fluid far from restraining boundaries is discussed, using ideas and generalizations taken over from our knowledge of turbulent flow of constant density, with the purpose of obtaining a criterion for the continued existence of fully turbulent flow. Naturally, the results can only be valid while the flow is fully turbulent, and the accuracy of the analysis depends on the occurrence of a breakdown in the mechanism of 'ordinary' turbulent flow before buoyancy forces become dominant.

The methods used are by no means new, and might be described as mutton (mixing-length theory), dressed as lamb by recasting it in terms of similarity hypotheses. The approach is somewhat similar to that used by Ellison (1957) who considered a boundary layer transmitting constant shear stress and constant heat flux. This is an example of the class of turbulent flows which have complete homogeneity in the direction of flow allied to essential inhomogeneity in the direction of shear. All such flows are restrained by the fluid boundaries, while the unrestrained flows considered in this paper are substantially homogeneous in the direction of shear and inhomogeneous in the direction of flow.

Although this paper is concerned primarily with flows in which radiative exchange of heat is negligible, the possibility of radiative transfer has been kept in mind and its effects will be considered in more detail in a following paper.

2. Conditions for similarity of the turbulent motion

The attempt to find the criterion which determines whether turbulent flow does or does not occur in a particular flow system is a part of the general problem of establishing the conditions under which flows with geometrically similar boundary conditions are dynamically similar, the independent non-dimensional parameters whose equality permits dynamical similarity losing one degree of freedom when they specify a marginally stable flow. With the earth's atmosphere in mind, we consider only flows of a nearly perfect gas, moving with velocity variations small compared with the local speed of sound, with temperature variations small compared with the absolute temperature, and with a length scale small compared with the scale height of the atmosphere. If effects of the earth's rotation are negligible, it may be shown (Batchelor 1953) that dynamical similarity of flows with geometrically similar boundary conditions, defined quantitatively by the three scales u_0, l_0, θ_0, referring to velocity variation, length, and potential temperature respectively, depends on equality of the three parameters $u_0 l_0/\nu$ (Reynolds number), $g\theta_0 l_0/(T_0 u_0^2)$ (Richardson number), ν/κ (Prandtl number), and equality or irrelevance of parameters depending on the radiative characteristics of the gas. Here ν is the kinematic viscosity, κ the thermometric conductivity, and T_0 the mean absolute temperature. For flows of essentially the same gas the Prandtl number does not vary, and experience of shear flows of high Reynolds numbers shows that viscosity has no effect on the large-scale components of the motion which contain nearly all the energy. It follows that similarity depends only on the scale Richardson number and on the radiation parameters.

No further simplification can be made by dimensional analysis alone without considering the dynamics of the flow. To the approximation implied by the restrictions set out above, the Navier–Stokes equations of motion are

$$\frac{D(U_i+u_i)}{Dt} = \frac{g_i}{T}\theta - \frac{1}{\rho}\frac{\partial(P+p)}{\partial x_i} + \nu\nabla^2(U_i+u_i), \tag{2.1}$$

and the continuity equation is

$$\frac{\partial(u_e+U_e)}{\partial x_e} = 0, \tag{2.2}$$

where U_i and u_i are the i-components of the mean fluid velocity and its fluctuation, θ is the temperature fluctuation about the mean T, P and p are the mean, and the fluctuation about the mean, of the pressure difference from the 'hydrostatic' pressure P_0 defined by $\partial P_0/\partial x_i = -\rho g_i$, $-g_i$ is the acceleration vector of the gravitational field, ρ is the mean density.

To this approximation, the direct effects of density variations on the velocity field are completely represented by the bouyancy term $g_i\theta/T$, although the distribution of the buoyancy forces will depend on the interaction of the turbulent motion and the heat sources of the flow. The action of these buoyancy forces may be described in several ways but a fundamental effect is the addition or subtraction of energy from the turbulent motion. The equation for the turbulent kinetic energy per unit mass is

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\overline{q^2}) + \overline{u_i u_e}\frac{\partial U_i}{\partial x_e} + \frac{\partial}{\partial x_e}\left(\overline{\tfrac{1}{2}q^2 u_e} + \frac{1}{\rho}\overline{pu_e}\right) + U_e\frac{\partial(\tfrac{1}{2}\overline{q^2})}{\partial x_e} = \frac{g_i}{T}\overline{\theta u_i} + \overline{\nu u_i\nabla^2 u_i}, \tag{2.3}$$

where $q^2 = u_i u_i$ and repeated suffices indicate summation over the three possible values of the repeated suffix. On the left-hand side of this equation, the first term is zero if the flow is statistically steady and the third and fourth terms represent transport of mechanical energy from one part of the flow to another, repectively transport by diffusive movements and through advection by the mean flow. Diffusive transport is usually effective only in the direction at right angles to the mean flow and can be eliminated from the equation by integrating over a whole section of the flow. Energy transport by advection cannot be eliminated in this way and it is of importance in all developing flows, e.g. boundary layers, jets and mixing layers, but its effects cannot usefully be considered without specifying the flow and it will be neglected in the following discussion. For this reason the results obtained apply strictly only to non-developing flows.

Three terms now remain in equation (2.3), they represent energy production by transfer from the mean flow and energy loss through work done against buoyancy forces and through viscous dissipation. To the extent that energy sources control its nature, the flow depends on the relative magnitudes of these three terms, or the ratio of the total loss of energy by buoyancy forces,

$$-\int \frac{g_i}{T}\overline{\theta u_i}\,dA,$$

to the total production by transfer from the mean flow,

$$-\int \overline{u_i u_e}\frac{\partial U_i}{\partial x_e}\,dA,$$

where the integrals extend over a whole section of the flow. This ratio,

$$[R_f]_I = \int \frac{g_i}{T}\overline{\theta u_i}\,dA \bigg/ \int \overline{u_i u_e}\frac{\partial U_i}{\partial x_e}\,dA, \tag{2.4}$$

is the flux Richardson number for the whole section of the flow. The ordinary flux form of the Richardson number,

$$R_f = \frac{g_i}{T}\overline{\theta u_i} \bigg/ \left[\overline{u_i u_e}\frac{\partial U_i}{\partial x_e}\right], \tag{2.5}$$

is the relevant *local* parameter only if turbulent transport of energy can be neglected. This analysis has been set out here to show that $[R_f]_I$ describes the relative effects of inertial and buoyancy forces on the turbulent motion, whether or not radiative transfer is an important element in the problem.

Since radiative transfer plays no direct part in the dynamics of the turbulent motion or in the turbulent energy balance, it is a plausible assumption that the flux Richardson number will describe the *motion* with or without radiative transfer of heat. It does *not* follow that the condition for the maintenance of turbulent motion must be expressible as a single critical value of the flux Richardson number. Obviously the flux Richardson number cannot exceed one (turbulent dissipation of energy cannot change sign) but laboratory observations of flows unaffected by radiative transfer indicate a critical value of the flux Richardson number substantially less than one. The critical condition appears to arise from a failure to achieve equilibrium in the interactions between the temperature field and the turbulent motion, which may occur in the following way. In a flow with given gradients of mean temperature and velocity, production of turbulent energy is proportional to the Reynolds stress and approximately proportional to the turbulent intensity. Loss of energy through buoyancy forces is proportional to the vertical heat flux or, in the absence of radiation, to the square-root of the turbulent intensity. Turbulent dissipation of energy is proportional to the $\frac{3}{2}$-power of the intensity. Clearly the ratio of the total energy loss to the energy production, considered as a function of the turbulent intensity, has a minimum value, and if this is greater than one no equilibrium intensity exists and the turbulence must decay. Introduction of radiative heat transfer into the system alters the dependence of turbulent heat transport on turbulent intensity and will lead to a change in the critical condition. Although the motion itself may be still determined by the flux Richardson number, the range of possible Richardson numbers will depend on the intensity of the radiative transfer. The next sections are concerned with the quantitative expression of these notions.

3. The turbulent heat transfer

To the approximation being used, the equation for the temperature of a fluid element is

$$\frac{D(T+\theta)}{Dt} + (U_i + u_i)\frac{g_i}{c_p} = k\nabla^2(T+\theta) + \frac{\mathscr{R}}{\rho c_p}, \tag{3.1}$$

where $\mathscr{R}$ is the net rate of heat gain by radiative processes. If this equation is multiplied by the temperature fluctuation and the mean value taken, an equation for the intensity of the temperature fluctuations is obtained,

$$\frac{\partial}{\partial t}(\tfrac{1}{2}\overline{\theta^2})+\overline{u_e\theta}\left(\frac{\partial T}{\partial x_e}+\frac{g_e}{c_p}\right)+U_e\frac{\partial(\frac{1}{2}\overline{\theta^2})}{\partial x_e}+\frac{\partial}{\partial x_e}(\tfrac{1}{2}\overline{\theta^2 u_e}) = \kappa\overline{\theta\nabla^2\theta}-\beta\overline{\theta^2}. \quad (3.2)$$

In this equation, the radiation term $\overline{\mathscr{R}\theta}/(\rho c_p)$ has been written as $-\beta\overline{\theta^2}$, where β may be regarded as the logarithmic rate of cooling of a fluid element by radiation alone. This radiation term is in general a function of the whole temperature field and its exact value will be considered in a second paper.

From equation (3.2), an estimate of the heat transport by convective movements can be obtained. From this point on, it is convenient to consider a stationary flow with vertical gradients of the horizontal mean velocity and of temperature. In addition, the transport terms will be omitted from the equations for the turbulent energy and for the intensity of the temperature fluctuations, which means that advection is neglected and that the quantities in the equations are suitably defined mean values for the whole flow. Using the ordinary notation with the Oz axis vertical and mean velocities along the Ox axis, equation (3.2) for $\frac{1}{2}\overline{\theta^2}$ becomes

$$\overline{w\theta}\left(\frac{\partial T}{\partial z}+\frac{g}{c_p}\right) = -\tfrac{1}{3}\overline{\theta^2}\frac{(\overline{w^2})^{1/2}}{L_\theta}-\beta\overline{\theta^2}, \quad (3.3)$$

where the conduction term $\kappa\overline{\theta\nabla^2\theta}$ has been replaced by $-\frac{1}{3}\overline{\theta^2}(\overline{w^2})^{1/2}/L_\theta$. This substitution is to some extent formal, but it expresses also the experimentally established result that the large-scale properties of turbulent flows are independent of the magnitudes of the viscosity and conductivity of the fluid. In constant density flows, the length L_θ is nearly equal to the integral scale of the turbulent motion. From this equation,

$$(\overline{\theta^2})^{1/2} = k_\theta(\overline{w^2})^{1/2}\frac{|\partial T/\partial z+g/c_p|}{\frac{1}{3}(\overline{w^2})^{1/2}/L_\theta+\beta}, \quad (3.4)$$

where

$$k_\theta = |\overline{w\theta}|/[\overline{w^2}\,\overline{\theta^2}]^{1/2},$$

and the heat transfer by turbulent convection is

$$\overline{\theta w} = -\frac{k_\theta^2}{k_u}|\overline{uw}|\frac{\partial T/\partial z+g/c_p}{\frac{1}{3}(\overline{w^2})^{1/2}/L_\theta+\beta}, \quad (3.5)$$

where $k_u = \overline{uw}/\overline{w^2}$. The flux Richardson number, defined by equation (2.5), is

$$R_f = \frac{k_\theta^2}{k_u}\frac{(g/T)(\partial T/\partial z+g/c_p)}{|\partial U/\partial z|(\frac{1}{3}(\overline{w^2})^{1/2}/L_0+\beta)}. \quad (3.6)$$

An expression for the flux Richardson number may be obtained in a form involving only gradients of mean values and some non-dimensional ratios. This is done by using the equation for the kinetic energy of the velocity fluctuations which is, omitting energy transport terms in the same way as transport terms were omitted in equation (3.3),

$$-\overline{uw}\frac{\partial U}{\partial z} = -\frac{g}{T}\overline{\theta w} + \frac{(\overline{w^2})^{3/2}}{L_\varepsilon} \tag{3.7}$$

where L_ε is the dissipation length scale and is nearly equal to the integral scale in constant density flows. Combining this equation with (3.5) leads to a quadratic equation for the turbulent heat transport, whose solution may be written in the non-dimensional form

$$R_f = \frac{(g/T)\overline{\theta w}}{\overline{uw}\,\partial U/\partial z} = \tfrac{1}{2}H\left[1-\left(1-12\frac{L_\theta}{L_\varepsilon}\frac{k_\theta^2}{k_u^2}\frac{R_i}{H^2}\right)^{1/2}\right], \tag{3.8}$$

where

$$H = 1 + \frac{3}{k_u}\frac{L_\theta}{L_\varepsilon}\frac{\beta}{|\partial U/\partial z|} \tag{3.9}$$

and

$$R_i = \frac{g}{T}\left(\frac{\partial T}{\partial z}+\frac{g}{c_p}\right)\bigg/\left(\frac{\partial u}{\partial z}\right)^2.$$

The number H is a measure of the ratio of the logarithmic rate of radiative cooling of a fluid element to the mean rate of shear. In the absence of radiative effects, it equals one.

4. Conditions for the maintenance of turbulent motion

In the previous section, an equation was obtained relating the turbulent heat transport to the gradients of mean velocity and potential temperature, to the radiative properties of the fluid, and to some non-dimensional ratios describing the turbulent motion. The validity of conclusions drawn from this equation depends only on further assumptions about the variation of the non-dimensional ratios with stability, since it is derived directly from the equations of motion and of heat. Neither theory nor experiment gives firm guidance on this point, and it will be assumed that the ratios L_θ/L_ε, k_u and k_θ, are nearly independent of stability. The first of these ratios, L_θ/L_ε, is simply the ratio of the logarithmic dissipation rates for temperature and velocity fluctuations. The large-scale fluctuations of temperature and vertical velocity are closely correlated and it seems unlikely that the rates of turbulent transfer down the scale of eddy sizes (which determine the magnitudes of the conductive and viscous dissipations) could be very dissimilar (Ellison (1957) makes an equivalent assumption for a constant-stress boundary layer). A more debatable assumption is that the shear coefficient k_u and the temperature–velocity correlation k_θ are nearly independent of stability, for there are circumstances in which it is certainly not true. In the last stages of decay of a turbulent mixing-layer between two streams of different temperature, most of the motion is in the form of gravity waves at the interface and permanent mixing on the molecular scale is a rarity. Both these coefficients then approach zero although at different rates. Again, in a thick boundary layer of constant stress and constant downward heat transport, buoyancy has a negligible influence near the ground and an overwhelming influence at great heights. Here, it is not possible to think of k_u and k_θ except as functions of height. The previous analysis has assumed stationary, non-developing flow and the use of typical or mean values of the velocity and temperature gradients implies that the motion in all parts of the flow is essentially similar, so both these sets of conditions are excluded. If the flow is everywhere characteristically turbulent, it is probable that the correlation factors are always large and near their values in constant density flows.

Returning now to the consideration of equation (3.8), we see that real values of the turbulent heat flux and the flux Richardson number are only possible if

$$R_i < \frac{1}{12} \frac{k_u^2}{k_\theta^2} \frac{L_\varepsilon}{L_\theta} \tag{4.1}$$

assuming, as we do from now, negligible effects of radiative transfer. Accepting the substantial constancy of k_u, k_θ and L_ε/L_θ, this sets the limit to possible values of the flux Richardson number,

$$R_f < \tfrac{1}{2}. \tag{4.2}$$

This limit to the possibility of turbulent motion arises from an impossibility of finding any real turbulent intensity to satisfy both the equation for the turbulent energy and the equation for the intensity of the temperature fluctuations, and it is additional to the limit set by the energy equation alone,

$$R_f < 1. \tag{4.3}$$

An interesting consequence of a limiting value for the flux Richardson number of one-half is that the turbulent intensity is finite in the critical flow. This can be seen by writing the energy equation (3.7) in the form

$$\frac{(\overline{w^2})^{1/2}}{k_u L_\varepsilon |\partial U/\partial z|} = 1 - R_f. \tag{4.4}$$

At the limit $R_f = \frac{1}{2}$, the turbulent intensity will be about one-quarter of the constant density value, and a sudden collapse of the turbulent motion occurs as the limit is passed.

5. Comparison with experiment

For a proper test of the validity of the analysis, it would be desirable to have experimental confirmation that the ratios k_u, k_θ, and L_θ/L_u, are nearly independent of stability, but practically no information is available. A few measurements of turbulent flows with stable density gradients do exist and these may be examined for conformity with the proposed criteria for turbulent motion ((4.1) and (4.2)), but nearly all these are of developing flows and in these stability is not a simple concept. In an unconstrained, developing flow, the Richardson number initially increases with distance from the origin of the flow but a comparatively rapid decay of the turbulent motion sets in downstream of the point where the Richardson number attains a critical value, this decay corresponding with the sudden collapse of the turbulent motion in a non-developing flow. As the decay is not instantaneous, the Richardson number may increase beyond the critical value. This makes the experimental determination of the critical value difficult and measurements of the kind to be discussed can do little more than indicate that the critical Richardson number may be near 0·1.

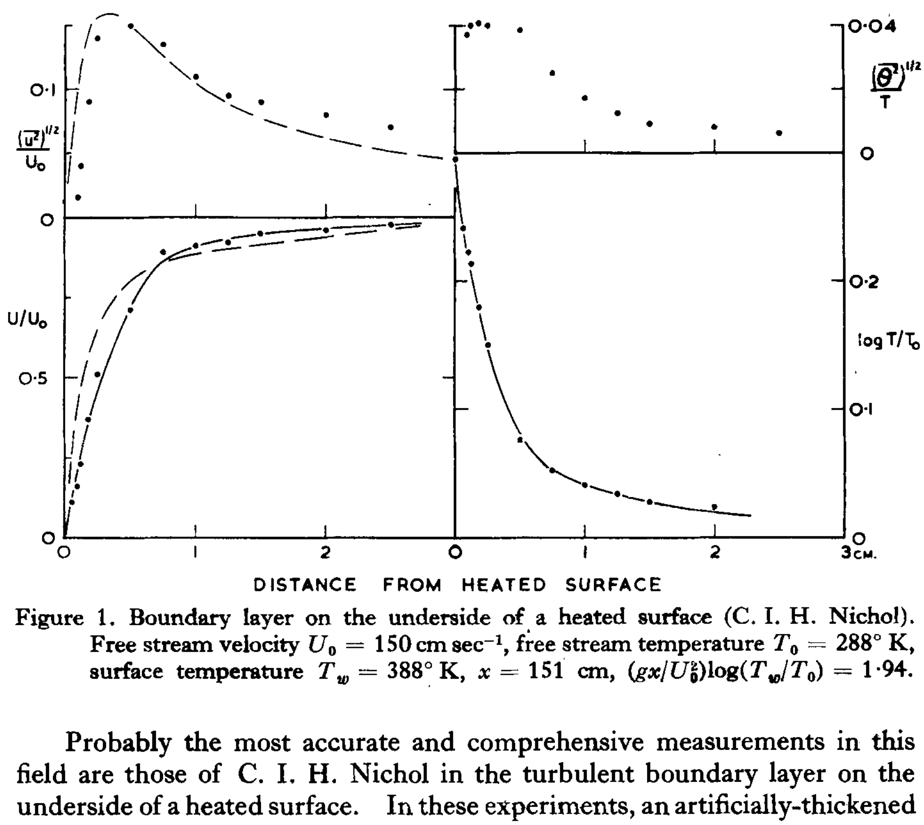

Figure 1. Boundary layer on the underside of a heated surface (C. I. H. Nichol). Free stream velocity $U_0 = 150\,\text{cm sec}^{-1}$, free stream temperature $T_0 = 288°$ K, surface temperature $T_w = 388°$ K, $x = 151$ cm, $(gx/U_0^2)\log(T_w/T_0) = 1{\cdot}94$.

Probably the most accurate and comprehensive measurements in this field are those of C. I. H. Nichol in the turbulent boundary layer on the underside of a heated surface. In these experiments, an artificially-thickened boundary layer was allowed to develop naturally for a distance of 127 cm, beyond which point the wall temperature was maintained at approximately 100° C above the temperature of the free stream. At an air-speed of $150\,\text{cm sec}^{-1}$, a nearly complete collapse of the turbulent motion was observed between 151 cm and 202 cm, the flow then having very low values of turbulent intensity and wall stress and a highly inflected form to the mean velocity profile (figures 1 and 2). This last effect is a consequence of the virtual disappearance of Reynolds shear stresses which earlier had kept the air next the wall moving. With their removal, this air slows down and the streamlines expand, displacing the outer flow. The beginnings of this process are clearly visible at $x = 151$ cm (figure 1), although the heat from the wall has not yet spread to the outer parts of the layer, and also in the measurements at an air-speed of 190 cm sec^{-1} (figure 3). As the individual measurements of velocity and temperature have random errors reflecting the considerable experimental difficulties, the computation of local Richardson numbers is not easy, as is shown by figure 4 in which are plotted local Richardson numbers computed by taking finite differences between neighbouring experimental points. The scatter is very large and only the most docile reader would agree from this that the mean values in the outer part of the layers are all less than 0·1. More representative numbers have been obtained by using the mean gradients over the outer parts of the layers, between 0·5 cm and 2·5 cm from the wall. These numbers are given in table 1 and may be more acceptable evidence that the Richardson number just before collapse is less than 0·1*.

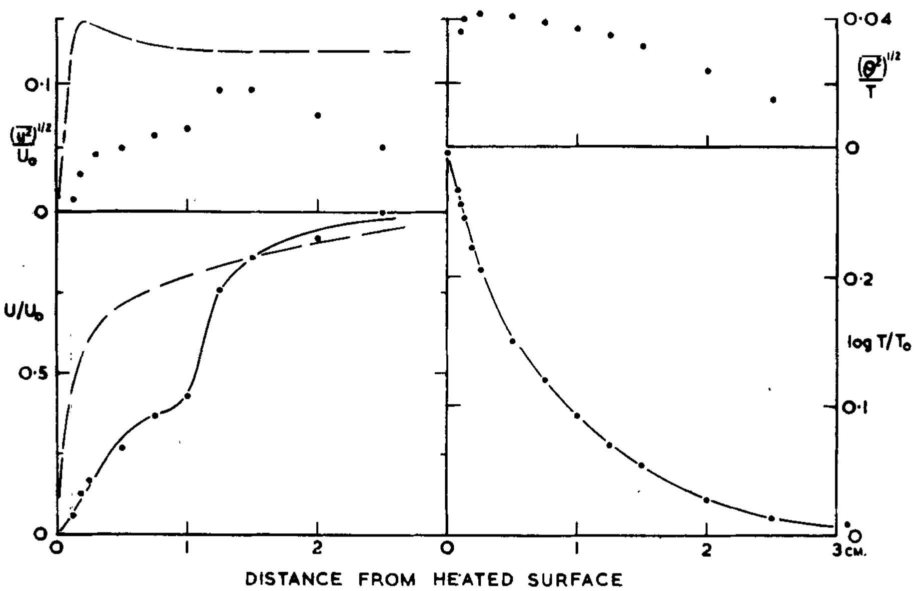

Figure 2. Boundary layer on the underside of a heated surface (C. I. H. Nichol). Free stream velocity $U_0 = 150$ cm sec^{-1}, free stream temperature $T_0 = 288°$ K, surface temperature $T_w = 388°$ K, $x = 202$ cm, $(gx/U_0^2)\log(T_w/T_0) = 2{\cdot}60$.

In the second series of experiments, a liquid jet was injected horizontally along the interface between a denser and less dense solution of salt in water. From a ciné record estimates could be made of the velocity and width of the jet. It was found that entrainment of fluid by the jet almost ceased when the Richardson number (which increases with distance from the nozzle) exceeded 0·3. Beyond this point, the velocity and cross-section of the jet remained nearly constant, indicating very little entrainment and

* It should be pointed out that this flow is essentially different from the constant-stress layer considered by Ellison (1957). The flow in this developing layer was not essentially different from that in one-half of the mixing layer between two streams.

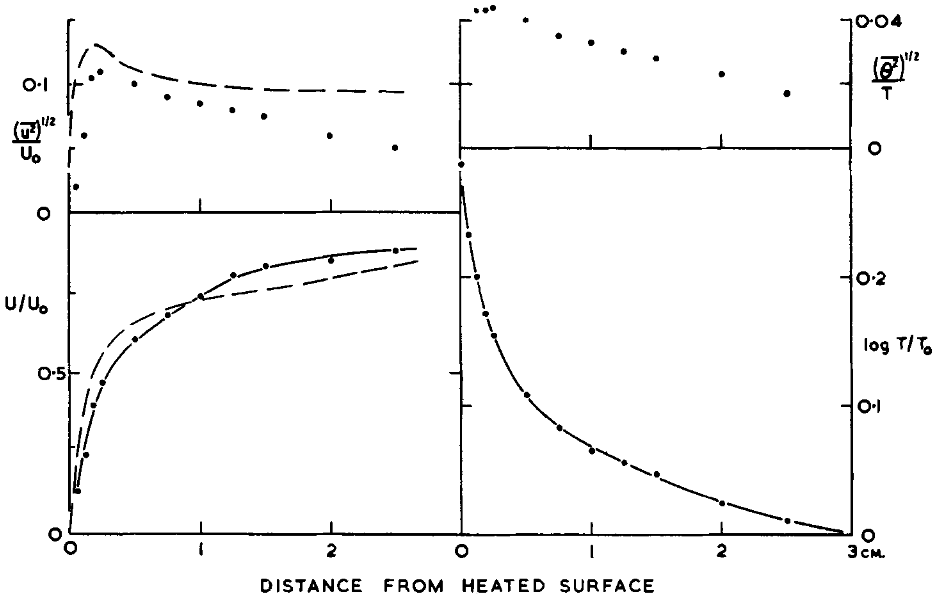

Figure 3. Boundary layer on the underside of a heated surface (C. I. H. Nichol). Free stream velocity $U_0 = 190$ cm sec^{-1}, free stream temperature $T_0 = 289°$ K, surface temperature $T_w = 385°$ K, $x = 202$ cm, $(gx/U_0^2)\log(T_w/T_0) = 1{\cdot}57$.

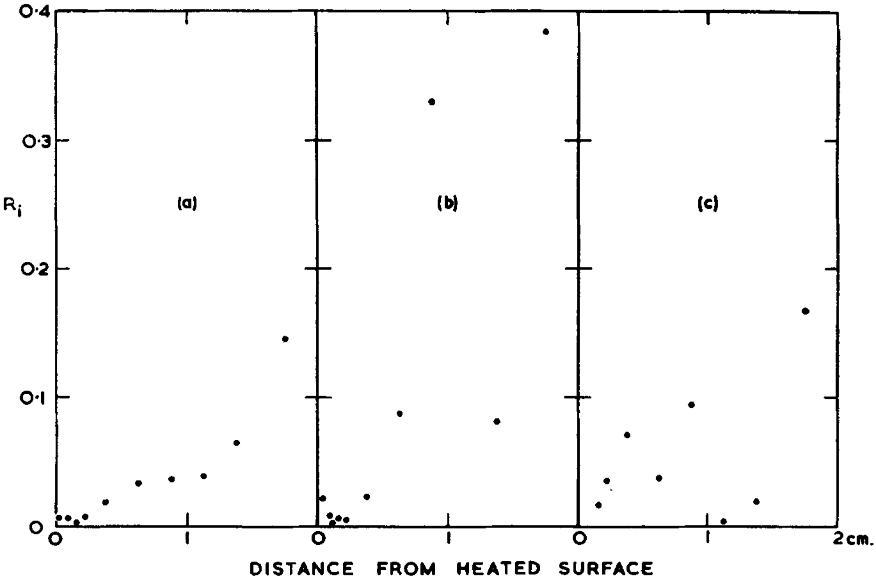

Figure 4. Boundary layer on the underside of a heated surface : local Richardson numbers.

(*a*) $U_0 = 150$ cm sec^{-1}, $x = 151$ cm,
(*b*) $U_0 = 150$ cm sec^{-1}, $x = 202$ cm,
(*c*) $U_0 = 190$ cm sec^{-1}, $x = 202$ cm.

presumably very little turbulent motion. The first signs of a diminution in rate of entrainment appeared at a Richardson number of about 0·05, and the critical number, in the sense used in the theory, is believed to lie between these limits (see figure 5).

Conditions	R_i	Remarks
$U_0 = 190$ cm sec^{-1} $x = 202$ cm, $T_w - T_0 = 96°$ C.	0·070	Collapse just beginning.
$U_0 = 150$ cm sec^{-1}, $x = 151$ cm, $T_w - T_0 = 100°$ C.	0·046	Spread of heat from the wall not yet complete.
$U_0 = 150$ cm sec^{-1}, $x = 202$ cm, $T_w - T_0 = 100°$ C.	0·022	Collapse nearly complete : note low value of R_i.

Table 1.

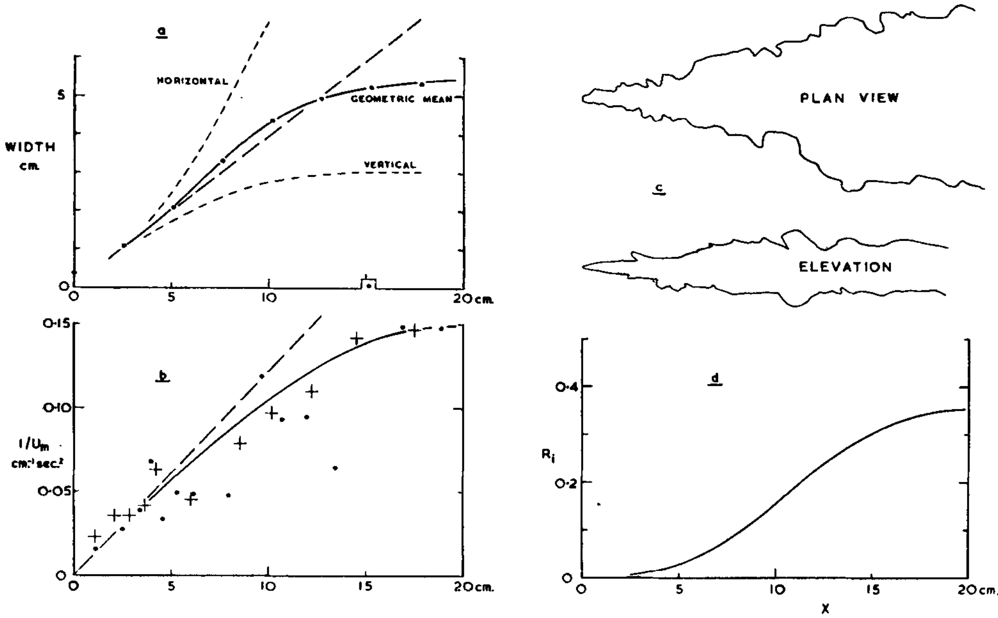

Figure 5. Spreading of a jet injected along the interface between saline solution of densities 1·032 and 1·125 gm cm^{-3}. (*a*) Variation of widths with distance from origin. The straight dashed line is the mean of observations of normal, constant-density jets. (*b*) Mean velocities of advance of marked fluid put into the jet. The dashed line is the mean of observations on a normal jet. (*c*) Instantaneous outlines of the jet, traced from enlargements of ciné records. The scale is the same as that of the horizontal scale of the other diagrams. (*d*) Mean Richardson numbers, computed as $R_i = 0{\cdot}064g[(\rho_1-\rho_2)/\bar{\rho}](D/U_m^2)$, where D is the observed width and U_m is the mean velocity of advance of dye.

The difficulty in all experiments with developing flows is that a failure of the regulating mechanism will cause the turbulence to decay but, while it decays, spreading of the flow and entrainment of ambient fluid continues. For this reason there is a lower critical number at which decay begins and an upper critical number attained when decay is complete. The lower number marks the failure of the normal regulating mechanism of the developing flow and should correspond with the one considered in the theory.

This conclusion that the critical Richardson number is about 0·08 appears to ignore the frequently quoted series of measurements of the flow of fresh-water of salt in the Kattegat, analysed by Taylor (1931) and others. In these measurements, Richardson numbers as high as one hundred were found although the shear stresses and transport of salinity were much larger than could be accounted for by viscosity and molecular diffusion. The observed values of the flux Richardson number were around 0·3. Some observations of a similar flow carried out by Dr J. S. Turner in the Cavendish Laboratory suggest that this apparent anomaly may be due to the non-existence of a critical Richardson number in the ordinary sense for a developing flow. In these experiments, fresh water was caused to flow over salt water which was coloured for easy identification. Near the beginning of the mixing zone, mixing is intense and caused by ordinary turbulent movements but, further downstream, the flow settles down to nearly laminar flow with a sharp interface between the fresh and the salt water. This interface is continuously distorted by irregular gravity waves which occasionally break, projecting coloured salt water upwards and, presumably, fresh water downwards. Most of these jets seem to fall back, losing only a small part of their volume by mixture but clearly transferring a considerable part of their momentum by the formation of 'inviscid' wakes of the vortex sheet type. Transfer of salinity is primarily due to molecular diffusion from the jets but is enhanced by the motion of the jets through the alien fluid. These qualitative considerations show that in this part of the flow 'turbulent' transport of salinity and momentum will each be considerably greater than the molecular rates, but that proportionately momentum transfer is very much more intense than salinity transport. It is probable that the Kattegat measurements were carried out under similar conditions and that the measured Richardson numbers do not refer to turbulent motion but to this random wave motion of the interface.

I am indebted to Mr C. I. H. Nichol for permission to abstract from his unpublished work the measurements that are represented in figures 1–4. I have benefited from observation and discussion of the experiments of Dr J. S. Turner on the flow of fresh water over salt water.

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